Comment on "Properties of the massive Thirring model from *XYZ* spin chain"

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It is shown that the continuum limit of the spin 1/2 Heisenberg *XYZ* model is far from sufficient for the site number of 16. Therefore the energy spectrum of the *XYZ* model obtained by Kolanovic *et al.* has nothing to do with the massive Thirring model, but it shows only the spectrum of the finite size effects.

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The spin 1/2 Heisenberg model is a rich theory, which contains a variety of field theories in the continuum limit. In particular, the Heisenberg *XYZ* model is proven to be equivalent to the massive Thirring model in the continuum limit.

Recently, Kolanovic, Pallua, and Prester [1] solved the Heisenberg *XYZ* model numerically to study the bound state problem in the massive Thirring model. In their paper, they claim that the binding energies of the bosonic states in the massive Thirring model are consistent with those of the semiclassical calculations by Dashen *et al.* [2], contrary to the predictions by Fujita *et al.* [3–5].

In this Comment, we show that the calculation of Kolanovic *et al.* [1] is far from reliable due to the rough resolution of their calculations. That is, the energy resolution in their calculation is not smaller than the mass parameter they used, and therefore there is no chance to obtain the binding energy of the system which should be in the continuum limit. The spectrum they obtain is nothing but some factor times the resolution $2\pi/L$.

Now, the spin 1/2 Heisenberg *XYZ* model can be described by the following Hamiltonian:

$$H=\sum_{i=1}^{N}\left(J_xS_i^xS_{i+1}^x+J_yS_i^yS_{i+1}^y+J_zS_i^zS_{i+1}^z\right), \tag{1}$$

where S_i^a is a spin operator at the site i. J_x, J_y, J_z denote the coupling constant, and N is the site number.

According to Luther [6], this Hamiltonian can be put into the following equations of motion which describe the massive Thirring model:

$$i\dot{\psi}_1(k)=v_0k\psi_1(k)-im_0\psi_2^\dagger(-k) -\frac{4J_z}{L}\sum_{k'}\psi_1(k-k')\rho_2(k'), \tag{2a}$$

$$i\dot{\psi}_2(k)=-v_0k\psi_2(k)+im_0\psi_1^\dagger(-k) -\frac{4J_z}{L}\sum_{k'}\psi_2(k-k')\rho_1(k'), \tag{2b}$$

where the fermion mass m_0 and v_0 are defined as

$$m_0=\frac{J_x-J_y}{2a}, \tag{3a}$$

$$v_0=\frac{1}{2}(J_x+J_y)-\frac{J_z}{2\pi}. \tag{3b}$$

Here, a denotes the lattice spacing constant, and the box length L is written as

$$L=Na. \tag{4}$$

The coupling constant g is related to J_x and v_0 as

$$g=\frac{2J_z}{v_0}. \tag{5}$$

Now, the value of N in the calculation of Ref. [1] is

$$N=16.$$

Since $L=(J_x-J_y)(N/2m_0)$, the resolution of the calculated spectrum becomes

$$\frac{2\pi}{L}=\frac{4\pi m_0}{(J_x-J_y)N}. \tag{6}$$

This is just comparable to the mass parameter m_0 in their calculations as shown in Table I. Thus it is impossible to extract any information on the bound state energy in the

TABLE I. The values of m_0 and $2\pi/L$ in units of $1/a$. In Ref. [1], $N=16$ is taken.

N	$2\pi/L$	m_0
16	0.393	
100	6.28×10^{-2}	0.1
1000	6.28×10^{-3}	

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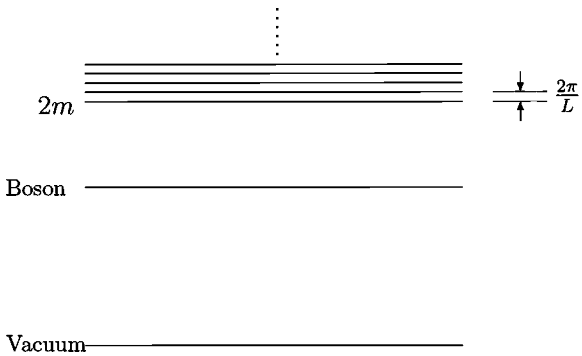

FIG. 1. A typical energy spectrum of the massive Thirring model with the box length L. See Ref. [4].

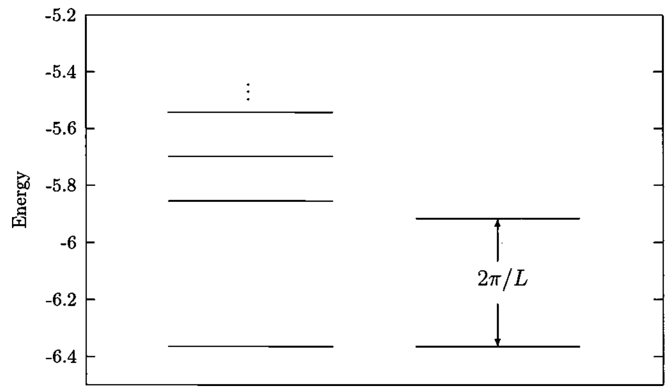

FIG. 2. The energy spectrum of several lowest states in the Heisenberg *XYZ* model with $m_0 = 0.5$, $N = 14$, $J_z = 0.9$, and $2\pi/L = 0.4488$ in units of $1/a$.

massive Thirring model. In order to obtain some reasonable results on the continuum version of the Heisenberg *XYZ* model, one has to satisfy the condition

$$\frac{2\pi}{L} \ll m_0 \ll \frac{2\pi}{L} N. \tag{7}$$

This suggests that if one wants to obtain any reliable information on the bound state of the massive Thirring model, one has to have the site number N which is at least larger than $N = 1000$.

In case N is small, then one obtains the spectrum which is just some factor times the resolution $2\pi/L$. This point is illustrated in Figs. 1 and 2. First, we show in Fig. 1 the calculation of the bound state spectrum in the massive Thirring model with the resolution of $2\pi/L$ [4]. There, one sees that the excited states above the free fermions are continuum states which are measured in units of $2\pi/L$. In Fig. 2, we show the calculated results of the spectrum with $N = 14$ site in the Heisenberg *XYZ* model. Even though their calculations are carried out with $N = 16$, there is practically no difference between $N = 14$ and $N = 16$ cases.

Here, we should comment on the lattice calculations in general. In terms of the correlation length, one may calculate the ground state energy in the lattice calculations. In this case, the condition of the validity in the lattice calculations is that the correlation length must be much smaller than the box length, but must be much larger than the lattice spacing.

Here, however, one cannot discuss the energy difference between the lowest state and the excited states (including the continuum states) because the energy difference between the lowest state and the excited states should be much larger than the resolution of $2\pi/L$. This is just the condition we imposed.

Therefore the results of Ref. [1] cannot be very reliable for the bound state spectrum of the massive Thirring model. But, of course, it does not necessarily mean that the spectrum of Dashen *et al.* [2] is incorrect.

[1] M. Kolanovic, S. Pallua, and P. Prester, Phys. Rev. D **62**, 025021 (2000).
[2] R.F. Dashen, B. Hasslacher, and A. Neveu, Phys. Rev. D **11**, 3424 (1975).
[3] T. Fujita and A. Ogura, Prog. Theor. Phys. **89**, 23 (1993).
[4] T. Fujita, Y. Sekiguchi, and K. Yamamoto, Ann. Phys. (N.Y.) **255**, 204 (1997).
[5] T. Fujita, T. Kake, and H. Takahashi, Ann. Phys. (N.Y.) **282**, 100 (2000).
[6] A. Luther, Phys. Rev. B **14**, 2153 (1976).